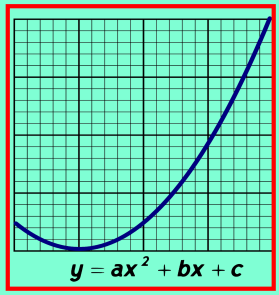
Math 125

Spring 2021

Lecture 17



Class QZ 13

Evaluate

Always

$$\begin{vmatrix} 2 & -5 & 1 \\ 1 & 3 & 0 \\ 3 & -2 & 1 \end{vmatrix} = 2\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} - (-5)\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix}$$

$$= 2(3 - 0) + 5(1 - 0) + 1(-2 - 9)$$

$$= 2(3) + 5(1) + 1(-11) = 6 + 5 - 11 = \boxed{0}$$

Rational exponents and radical notations:

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Radical

Radicand

index

Given $\sqrt[5]{x^3}$

1) Radicand $= x^3$

2) Index $= 5$

3) write using rational exponent

$x^{\frac{3}{5}}$

Given $(2x-3)^{\frac{1}{3}}$ ← index

1) write using radical notation $\sqrt[3]{(2x-3)^1} = \sqrt[3]{2x-3}$

2) Index $= 3$

3) Radicand $2x-3$

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when index is even $\Rightarrow$ Radicand ≥ 0

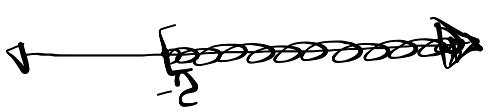
Answer ≥ 0

$\sqrt[4]{-16}$ is undefined. even index, radicand < 0

$\sqrt[6]{x+2}$ even index $\Rightarrow$ Radicand ≥ 0

$x+2 \geq 0$

$\boxed{x \geq -2}$



$[-2, \infty)$

$\sqrt[n]{x}$ = Answer $\Leftrightarrow$ Answer$^n = x$

If n is even $\Rightarrow$ $x \geq 0$, Answer ≥ 0

If n is odd $\Rightarrow$ x and Answer have Same Sign

both + or both −

when index = n not given

$\Rightarrow$ it is assumed to be 2

$\Rightarrow$ Square root

$\sqrt[3]{8} = 2$ Answer$^3 = 8$

$2^3 = 8$

$\sqrt[5]{-32} = -2$ Answer$^5 = -32$

odd index $(-2)^5 = -32$

$\sqrt[4]{-81}$ undefined

even index $\Rightarrow$ Radicand ≥ 0

$-81 \geq 0$ False

$\sqrt[4]{81} = \boxed{3}$

$(3)^4 = 81$

even Root

Radicand ≥ 0

Answer ≥ 0

$\sqrt[3]{-125} = \boxed{-5}$ $(-5)^3 = -125$ ✓

odd index

Radicand & Answer must have Same Signs.

$\sqrt{100} = 10$ $10^2 = 100$ ✓

even index (No index $\Rightarrow$ 2)

Radicand ≥ 0 ✓ Answer ≥ 0

Simplify, final Answer in a Single radical

$$\sqrt[4]{x} \cdot \sqrt[5]{x} \qquad x^m \cdot x^n = x^{m+n}$$

$$= x^{\frac{1}{4}} \cdot x^{\frac{1}{5}} = x^{\frac{1}{4}+\frac{1}{5}} = x^{\frac{9}{20}} = \boxed{\sqrt[20]{x^9}}$$

$$\sqrt[5]{x^2} \cdot \sqrt[3]{x} = x^{\frac{2}{5}} \cdot x^{\frac{1}{3}} = x^{\frac{2}{5}+\frac{1}{3}}$$

$$\frac{2\cdot 3}{5\cdot 3} + \frac{1\cdot 5}{3\cdot 5} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

$$= x^{\frac{11}{15}} = \boxed{\sqrt[15]{x^{11}}}$$

$$\frac{\sqrt[4]{x^3}}{\sqrt[5]{x^2}} = \frac{x^{\frac{3}{4}}}{x^{2/5}} = x^{\frac{3}{4}-\frac{2}{5}} = x^{\frac{7}{20}} = \boxed{\sqrt[20]{x^7}} \qquad \frac{x^m}{x^n} = x^{m-n}$$

Simplify

$$\frac{\sqrt{x} \cdot \sqrt[3]{x}}{\sqrt[4]{x} \cdot \sqrt[5]{x}} = \frac{x^{\frac{1}{2}} \cdot x^{\frac{1}{3}}}{x^{\frac{1}{4}} \cdot x^{\frac{1}{5}}}$$

$$= \frac{x^{\frac{1}{2}+\frac{1}{3}}}{x^{\frac{1}{4}+\frac{1}{5}}} = \frac{x^{\frac{5}{6}}}{x^{\frac{9}{20}}} = x^{\frac{5}{6}-\frac{9}{20}}$$

$$\frac{5\cdot 10}{6\cdot 10} - \frac{9\cdot 3}{20\cdot 3} = \frac{50-27}{60} = \frac{23}{60}$$

$$= x^{\frac{23}{60}} = \boxed{\sqrt[60]{x^{23}}}$$

LCD = 60

Index = 60

Radicand = x^{23}

Rules of radicals:

$$\boxed{\sqrt[n]{x^n} = x \quad , \left(\sqrt[n]{x}\right)^n = x}$$

Assume all radicands ≥ 0

$$\sqrt[n]{AB} = \sqrt[n]{A}\,\sqrt[n]{B}$$

$$\sqrt{20x} = \sqrt{4 \cdot 5x} = \sqrt{4}\,\sqrt{5x} = \boxed{2\sqrt{5x}}$$

index=3

$$\sqrt[3]{80x^4} = \sqrt[3]{8x^3 \cdot 10x} = \sqrt[3]{8x^3}\,\sqrt[3]{10x}$$

$$= \boxed{2x\sqrt[3]{10x}}$$

index=2

$$\sqrt{300x^5} = \sqrt{100 \cdot 3 \cdot x^2 \cdot x^2 \cdot x} = \sqrt{100\,x^2 x^2}\,\sqrt{3x}$$

$$= 10\,x\,x\,\sqrt{3x} = \boxed{10x^2\sqrt{3x}}$$

Simplify

$$\sqrt[3]{54x^6y^{12}}$$

Recall

$54 = 2 \cdot 27$

$= 2 \cdot 3^3$

$$= \sqrt[3]{27 \cdot 2 \cdot x^3 \cdot x^3 \cdot y^3 y^3 y^3 y^3}$$

$$= \sqrt[3]{\cancel{3^3}\, \cancel{x^3}\, \cancel{x^3}\, \cancel{y^3}\, \cancel{y^3}\, \cancel{y^3}\, \cancel{y^3}}\;\sqrt[3]{2}$$

$$= 3\,x\,x\,y\,y\,y\,y\,\sqrt[3]{2} = \boxed{3x^2y^4\sqrt[3]{2}}$$

Simplify $\sqrt[4]{16x^5y^{11}} = \sqrt[4]{2^4 \cdot x^4 \cdot x \cdot y^8 \cdot y^3}$

$$= \sqrt[4]{\cancel{2^4}\,\cancel{x^4}\,\cancel{y^8}}\;\sqrt[4]{x\,y^3}$$

$$= \boxed{2xy^2\sqrt[4]{xy^3}}$$

Simplify

$$\sqrt[3]{5000\,x^4\,y^{11}\,z^{17}}$$

$x^4 = x^3 \cdot x$

$y^{11} = y^9 \cdot y^2$

$z^{17} = z^{15} \cdot z^2$

$5000 = 1000 \cdot 5$

$= 10^3 \cdot 5$

$$= \sqrt[\cancel{3}]{10^{\cancel{3}}\,x^{\cancel{3}}\,y^9\,z^{15}}\;\sqrt[3]{5xy^2z^2}$$

$$= \boxed{10\,x\,y^3\,z^5\,\sqrt[3]{5x\,y^2z^2}}$$

Simplify

$$\sqrt[5]{-32\,x^7\,y^{11}\,z^{33}}$$

$-32 = (-2)^5$

$x^7 = x^5 x^2$

$y^{11} = y^{10} \cdot y$

$z^{33} = z^{30} z^3$

$$= \sqrt[\cancel{5}]{(-2)^{\cancel{5}}\,x^{\cancel{5}}\,y^{\cancel{10}\,2}\,z^{\cancel{30}\,6}}\;\sqrt[5]{x^2yz^3}$$

$$= \boxed{-2\,x\,y^2z^6\,\sqrt[5]{x^2yz^3}}$$

Distribute and Simplify

$50 = 1 \cdot 50$
$= 2 \cdot 25$
$= 5 \cdot 10$

$$\sqrt{5}\left(\sqrt{10} - \sqrt{5}\right)$$

$$= \sqrt{5}\sqrt{10} - \sqrt{5}\sqrt{5}$$

$$= \sqrt{50} - \sqrt{25} = \sqrt{25}\sqrt{2} - \sqrt{25}$$

$$= \boxed{5\sqrt{2} - 5}$$

Distribute and Simplify

$$2\sqrt{6}\left(3\sqrt{2} - 5\sqrt{6}\right)$$

$$= 2\sqrt{6} \cdot 3\sqrt{2} - 2\sqrt{6} \cdot 5\sqrt{6}$$

$$= 6\sqrt{12} - 10\sqrt{36}$$

$$= 6\sqrt{4}\sqrt{3} - 10 \cdot 6$$

4 is a perfect-square

$$= 6 \cdot 2\sqrt{3} - 60$$

$$= \boxed{12\sqrt{3} - 60}$$

Foil & Simplify

$$\left(\sqrt{5} + \sqrt{3}\right)\left(\sqrt{5} - \sqrt{3}\right)$$

$$= \sqrt{25} - \cancel{\sqrt{15}} + \cancel{\sqrt{15}} - \sqrt{9}$$

$$= 5 - 3 = \boxed{2}$$

Foil and Simplify

$$(2\sqrt{3}+1)^2 = (2\sqrt{3}+1)(2\sqrt{3}+1)$$

$$= 2\sqrt{3}\cdot 2\sqrt{3} + 2\sqrt{3}\cdot 1 + 1\cdot 2\sqrt{3} + 1$$

$$= 4\sqrt{9} + 2\sqrt{3} + 2\sqrt{3} + 1$$

$$= 4\cdot 3 + 4\sqrt{3} + 1$$

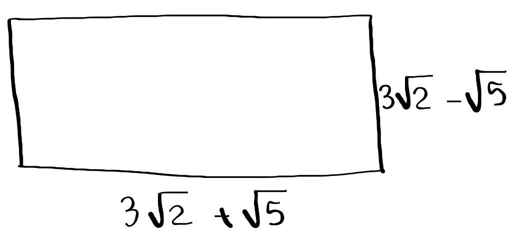
$$= 12 + 4\sqrt{3} + 1$$

$$= \boxed{13 + 4\sqrt{3}}$$

find area & perimeter of

$A = LW$

$P = 2L + 2W$



$$A = LW = (3\sqrt{2}+\sqrt{5})(3\sqrt{2}-\sqrt{5})$$

$$= 9\sqrt{4} \cancel{-3\sqrt{10}} \cancel{+3\sqrt{10}} - \sqrt{25} = 9\cdot 2 - 5$$

$$= 18 - 5 = \boxed{13}$$

sq. units.

$$P = 2L + 2W$$

$$= 2(3\sqrt{2}+\sqrt{5}) + 2(3\sqrt{2}-\sqrt{5}) =$$

$$= 6\sqrt{2} + 2\sqrt{5} + 6\sqrt{2} - 2\sqrt{5} = \boxed{12\sqrt{2}}$$ units

Simplify $(5\sqrt{2} - 3)^2 = (5\sqrt{2} - 3)(5\sqrt{2} - 3)$

$$= 25\sqrt{4} - 15\sqrt{2} - 15\sqrt{2} + 9$$

$$= 25 \cdot 2 - 30\sqrt{2} + 9$$

$$= \boxed{59 - 30\sqrt{2}}$$

Foil and Simplify

$$\left(\sqrt[3]{3} - \sqrt[3]{2}\right)\left(\sqrt[3]{9} + \sqrt[3]{6} + \sqrt[3]{4}\right)$$

$$= \sqrt[3]{27} + \cancel{\sqrt[3]{18}} + \cancel{\sqrt[3]{12}} - \cancel{\sqrt[3]{18}} - \cancel{\sqrt[3]{12}} - \sqrt[3]{8}$$

$$= 3 - 2 = \boxed{1}$$

Foil & Simplify

$$\left(\sqrt[3]{5} + \sqrt[3]{3}\right)\left(\sqrt[3]{25} - \sqrt[3]{15} + \sqrt[3]{9}\right)$$

$$= \sqrt[3]{125} - \cancel{\sqrt[3]{75}} + \cancel{\sqrt[3]{45}} + \cancel{\sqrt[3]{75}} - \cancel{\sqrt[3]{45}} + \sqrt[3]{27}$$

$$= 5 + 3 = \boxed{8}$$

$A+B$ and $A-B$ are called conjugate of each other.

Conjugate of $\sqrt{3}+1$ is $\sqrt{3}-1$.

Conjugate of $2\sqrt{5}-\sqrt{7}$ is $2\sqrt{5}+\sqrt{7}$

Multiply $\sqrt{10}-3$ by its Conjugate, then Simplify

$$(\sqrt{10}-3)(\sqrt{10}+3)$$

$$=\sqrt{100}+\cancel{3\sqrt{10}}-\cancel{3\sqrt{10}}-9$$

$$=10-9=\boxed{1}$$

Multiply $5\sqrt{2}+\sqrt{5}$ by its conjugate, then Simplify

$$(5\sqrt{2}+\sqrt{5})(5\sqrt{2}-\sqrt{5})$$

$$=25\sqrt{4}-\cancel{5\sqrt{10}}+\cancel{5\sqrt{10}}-\sqrt{25}$$

$$=25\cdot 2-5=\boxed{45}$$

Solve

$$\begin{cases} 2x^2 + y^2 = 33 \\ x^2 - 2y^2 = -46 \end{cases}$$

There are 4 possible answers.

$y^2 = 33 - 2x^2$

$x^2 - 2(33 - 2x^2) = -46$

$x^2 - 66 + 4x^2 = -46$

$5x^2 - 66 = -46$

$5x^2 = -46 + 66$

$5x^2 = 20$ Divide by 5

$x^2 = 4$ $\boxed{x = \pm 2}$

$y^2 = 33 - 2\boxed{x^2}$

$= 33 - 2(4)$

$y^2 = 33 - 8$

$y^2 = 25$ $\boxed{y = \pm 5}$

$\{(2,5), (2,-5), (-2,5), (-2,-5)\}$

Ordered Pair

~~$\{x = 2, y = 3\}$~~

~~$\{x = 1, y = 2, z = 3\}$~~

$(1, 2, 3)$ ✓

Ordered Triple

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y varies directly as x^4

k.

y is 64 when x is 2

Find y when x is -2.

$y = kx^4$

$64 = k(2)^4$

$64 = 16k$

$\boxed{k = 4}$

$y = 4x^4$

$y = 4(-2)^4$

$\boxed{y = 64}$

y varies inversely as 5th root of x.

K/

$$y = \frac{k}{\sqrt[5]{x}}$$

y is 10 when x is -32

$$10 = \frac{k}{\sqrt[5]{-32}}$$

Find y when x is 32.

$$10 = \frac{k}{-2}$$

$$\boxed{k = -20}$$

$$y = \frac{-20}{\sqrt[5]{x}} = \frac{-20}{\sqrt[5]{32}}$$

$$= \frac{-20}{2}$$

$$\boxed{y = -10}$$

Class QZ 14

Hint: Use Subs. Method

Solve

$$\begin{cases} x^2 + y^2 = 50 \\ y - x = 0 \end{cases}$$